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LOCAL INSTABILITY OF CENTRALLY LOADED COLUMNS OF CHANNEL SECTION AND Z-SECTION

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SUMMARY

Charts are presented for the coefficients in formulas for the critical compressive stress at which cross-sectional distortion begins in a thin-wall member with either a channel section or a Z-section with identical flanges. The energy method of Timoshenko was used in the theoretical calculations required for the construction of the charts. The deflection equations were carefully selected to give good accuracy.

The calculation of the critical compressive stress at stresses beyond the elastic range is briefly discussed. In order to demonstrate the use of the formulas and the charts in engineering calculations, two illustrative problems are included.

INTRODUCTION

In the design of compression members for aircraft, whether they be stiffeners in stressed-skin structures or struts in trussed structures, the allowable stress for the member is equal to the lowest strength corresponding to any of the possible types of failure. In references 1 and 2, all types of column failure are classed under two headings:

(a) Primary, or general, failure.

(b) Secondary, or local, failure.

Primary, or general, failure of a column is defined as any type of failure in which the cross sections are translated, rotated, or both translated and rotated but not distorted in their own planes (fig. 1). Secondary, or local, failure of a column is defined as any type of failure in which the cross sections are distorted in their own planes but not translated or rotated (fig. 2). Consideration is given in this paper only to local failure.

One of the factors to be considered in a study of local failure is the critical compressive stress at which the cross section begins to distort. This critical stress can usually be given in coefficient form. The purpose of this paper is to present charts that will be useful in establishing the coefficient to be used in the calculation of the critical compressive stress at which cross-sectional distortion begins in a thin-wall channel section or Z-section with identical flanges.

The energy method of Timoshenko was used for the calculations required to evaluate the coefficient plotted in the charts. (See reference 3.) The calculations, which are long and were made as a part of a more extended study of local failure in thin-metal columns, have been omitted from this paper.

This paper is the second of a series on the general subject of local failure in thin-metal columns. The first report of the series (reference 4) is concerned with local failure in thin-wall rectangular tubes.

Bernard Rubenstein, formerly of the N.A.C.A. staff, performed all the mathematical derivations required for the preparation of this paper.

CHARTS

The calculation of the critical compressive stress at which cross-sectional distortion begins in a channel section or a Z-section is, in reality, a problem in the buckling of thin plates, proper consideration being given to the interaction between adjacent plates composing the cross section. For the columns of channel section and Z-section considered in this paper, the flanges have identical dimensions. The conditions of symmetry in the cross section require that, when one flange buckles, the other flange also buckles. (See fig. 2.) Thus, the channel section and the Z-section consist of two basic plate elements, i.e., flange plates and a web plate.

Timoshenko has given the critical stress for a rectangular plate under edge compression in the following form (reference 5, p. 605):

$$f_{cr} = \frac{k \pi^2 E t^2}{12 (1 - \mu^2) b^2}$$

where

- E is tension-compression modulus of elasticity for the material.
- μ, Poisson's ratio for the material.
- t, thickness of the plate.
- b, width of the plate.
- k, a nondimensional coefficient that depends upon the conditions of edge support and the dimensions of the plate.

This equation can be used to calculate the critical compressive stress at which cross-sectional distortion begins in channel- and Z-section columns. If t and b are the thickness and the width, respectively, of the flange, then the restraining effect of the web, whether positive or negative, is included in the coefficient k. If t and b refer to the thickness and the width, respectively, of the web, then the restraining effect of the flange, whether positive or negative, is also included in the coefficient k but a different set of values for k is obtained. It is therefore necessary to decide whether t and b in the equation for the critical stress shall refer to the dimensions of the flange or to the dimensions of the web. In certain limiting cases, one form of the equation is to be preferred; whereas, in other limiting cases, the other form is preferable. In this report, both forms of the equation will be given, either of which may be used to calculate the critical stress for channel sections and Z-sections. For the flange plate,

$$f_{cr} = \frac{k_F \pi^2 E t_F^2}{12 (1 - \mu^2) b_F^2} \tag{1}$$

For the web plate,

$$f_{cr} = \frac{k_W \pi^2 E t_W^2}{12 (1 - \mu^2) b_W^2} \tag{2}$$

where

t_F and t_W are the thickness of the flange and the web plates, respectively.

b_F and b_W, the width of the flange and the web plates, respectively.

k_F and k_W, nondimensional coefficients that depend upon the dimensions of the channel section or the Z-section. (See figs. 3 and 4.)

The curves given in figures 3 and 4 were obtained by plotting the calculated values of k_F and k_W given in tables I and II, respectively. These values were computed by the energy method previously mentioned.

The relation between k_F and k_W for a given channel section or Z-section is sometimes of interest. This relation is obtained by equating the right sides of equations (1) and (2). Thus

$$k_F \frac{t_F^2}{b_F^2} = k_W \frac{t_W^2}{b_W^2}$$

from which

$$k_F = \left(\frac{b_F}{b_W}\right)^2 \left(\frac{t_W}{t_F}\right)^2 k_W \qquad (3)$$

LIMITATIONS OF CHARTS

The charts in figures 3 and 4 may be regarded as close approximations, the errors being not more than about 1 percent. The values of k_F and k_W given in the charts are the minimum values possible for a channel- or a Z-section column of infinite length. For engineering use, however, these values will apply to any channel- or Z-section column having a length greater than about twice the width of the web or the flange, depending on which is the wider. The length of all members likely to be encountered in aircraft design will thus fall within the range to which figures 3 and 4 apply. It should be mentioned that, for very short columns of channel section or Z-section where the length does have an appreciable effect, the values of the coefficient are conservative.

The values of k_F and k_W given herein apply to columns with channel section or Z-section in which the material is both elastic and isotropic. Steel, aluminum alloys, and other metallic materials usually satisfy these conditions provided that the material is not stressed beyond the elastic range. When a material is stressed beyond the proportional limit in one direction, it is no longer elastic and is probably no longer isotropic. In a later portion of this paper, the use of equations (1) and (2) is shown in the calculation of the critical stress when the columns are loaded beyond the proportional limit.

DEFLECTION EQUATIONS

The deflection equations used in the energy solution are: For the flanges,

$$w_F = \left\{A \frac{y_F}{b_F} - \frac{B}{3.889} \left[\left(\frac{y_F}{b_F}\right)^5 - 4.963 \left(\frac{y_F}{b_F}\right)^4 + 9.852 \left(\frac{y_F}{b_F}\right)^3 - 9.778 \left(\frac{y_F}{b_F}\right)^2\right]\right\} \sin \frac{n\pi x}{L} \tag{4}$$

For the web,

$$w_W = \left[40 \frac{y_W}{b_W} \left(1 - \frac{y_W}{b_W}\right) + D \sin \frac{\pi y_W}{b_W}\right] \sin \frac{n\pi x}{L} \tag{5}$$

where

w_F and w_W are deflection normal to flange and web, respectively.

L, length of member.

n, number of half-waves that form in the length L. The ratio L/n is therefore the half-wave length of a wrinkle in the direction of the length.

b_F and b_W, width of flange and web, respectively.

x, coordinate measured from end of member.

y_F and y_W, coordinates measured from one corner in the direction of flange and web, respectively.

A, B, C, and D, arbitrary deflection amplitudes. The values of A and B for the flanges are expressed in terms of C and D for the web through the use of the conditions that the corner angles are maintained during buckling and that the bending moments at the corner are in equilibrium. The values of D/C and L/n are then given values that cause the critical stress to be a minimum.

The foregoing deflection equations used in the energy solution were carefully selected. Although no direct calculation of the error has been made, it is believed that the values of k_F and k_W are correct to within a fraction of 1 percent. This belief is justified because, in the limiting cases for which exact solutions are available, the precision is within these limits. In addition, other problems in which these deflection equations have been used gave a precision better than 1 percent.

If $B = C = 0$, the deflection equations (4) and (5) reduce to the same equations used by Parr and Beakley (reference 6) in their study of local instability (plate failure) of channel columns.

DISCUSSION OF CHARTS

Figure 3 gives values of k_F plotted against b_W/b_F for values of $t_W/t_F = 0.5$, 1, and 2. When the web is very narrow in comparison with the flanges (b_W/b_F small), the flanges are weaker than the web. As b_W/b_F increases, a point is reached where the web becomes the weaker part of the cross section. This point is clearly discernible for $t_W/t_F = 0.5$ and 2 in figure 3 where these curves break sharply at $b_W/b_F = 1.8$ and 3.3, respectively.

Figure 4 gives values of k_W plotted against b_F/b_W for the same values of t_W/t_F. When the flanges are very narrow in comparison with the web (b_F/b_W small), the web is weaker than the flanges. As b_F/b_W increases, a point is reached where the flange becomes the weaker part of the cross section. This point is clearly discernible for $t_W/t_F = 0.5$ in figure 4 where this curve breaks sharply at $b_F/b_W = 0.55$.

CRITICAL STRESS FOR LOADING BEYOND THE PROPORTIONAL LIMIT

In the elastic range, the critical compressive stress for an ordinary column that fails by bending is given by the Euler formula. Beyond the proportional limit that marks the upper end of the elastic range, the reduced slope of the stress-strain curve requires that an effective modulus $\overline{E}$ be substituted for Young's modulus E in the Euler formula. The value of $\overline{E}$ is sometimes written as τE,

$$\overline{E} = \tau E \tag{6}$$

The value of the nondimensional coefficient τ varies with stress. By the use of the double-modulus theory of column action, theoretical values of τ can be obtained from the compressive stress-strain curve for the material (reference 5, p. 572, and references 7 and 8). Tests show that, in practice, theoretical values of τ, derived on the assumption that no deflection occurs until the critical load is reached, are too large. The value of τ for practical use is best obtained from the accepted column curve for the material in the manner outlined in the illustrative problem of reference 4. The values of τ thus obtained take into account the effect of imperfections that cause deflection from the beginning of loading as well as other factors that may have a bearing on the strength.

For cross-sectional distortion of a thin-wall column of channel section or Z-section, the critical compressive stress in the elastic range is given by either equation (1) or equation (2). Beyond the proportional limit, the critical compressive stress is given by these equations with an effective modulus ηE substituted for Young's modulus E or: For the flange plate,

$$f_{cr} = \frac{\eta\, k_F\, \pi^2\, E\, t_F^{\,2}}{12\,(1 - \mu^2) b_F^{\,2}} \tag{7}$$

For the web plate,

$$f_{cr} = \frac{\eta\, k_W\, \pi^2\, E\, t_W{}^2}{12\,(1 - \mu^2) b_W{}^2} \tag{8}$$

In the absence of adequate test data, the value of the non-dimensional coefficient η cannot be definitely established. It is reasonable to suppose, however, that η and τ are related in some way.

Various equations relating η and τ have been suggested. The discussion of reference 4 points out that, when η is considered to be a function of τ, the equation for η will depend upon the manner of evaluation of τ. If τ is determined from the stress-strain curve on the assumption that no deflection takes place until the critical stress is reached, the effect of deflections from the beginning of loading must be separately considered. If τ is determined from the accepted column curve for the material in the manner outlined in the illustrative problems of reference 4, part, if not all, of this effect is automatically considered.

A careful study of the theory and of such experimental data as are available indicates that a conservative assumption is

$$\eta = \frac{\tau + 3\sqrt{\tau}}{4} \tag{9}$$

provided that τ is evaluated by use of the accepted column curve for the material. Equation (9) will probably have to be modified, however, as more test data become available.

Now τ is itself a function of the critical stress f_{cr}. Hence η is a function of f_{cr}. Consequently, equations (7) and (8) cannot be solved directly for f_{cr}. If each equation is divided by η, however, f_{cr}/η is given directly by the geometrical dimensions of the cross section and the charts of figures 3 and 4. For the flange plate,

$$\frac{f_{cr}}{\eta} = \frac{k_F\, \pi^2\, E\, t_F{}^2}{12\,(1 - \mu^2) b_F{}^2} \tag{10}$$

For the web plate,

$$\frac{f_{cr}}{\eta} = \frac{k_W \pi^2 E t_W^2}{12 (1 - \mu^2) b_W^2} \tag{11}$$

The relation between f_{cr} and f_{cr}/η can be determined from a knowledge of the column curve for the material, as outlined in the illustrative problem of reference 4. In figure 5, several such curves are given for 24ST aluminum alloy for different assumed relations between η and τ. When the value of f_{cr}/η has been obtained by use of equation (10) or equation (11), the value of f_{cr} is read from the appropriate curve of figure 5.

The ultimate strength of a thin-wall column of channel section or Z-section will, in general, be higher than the load at which cross-sectional distortion begins. At stresses approaching the yield point of the material, the critical load and the ultimate load approach the same value. No attempt has been made in this paper to discuss the ultimate strength of a thin-wall column of channel section or Z-section; the solution for the critical load logically precedes the solution for the ultimate load.

ILLUSTRATIVE PROBLEM

It is desired to calculate the critical compressive stress at which cross-sectional distortion begins in two channel columns constructed of 24ST aluminum alloy:

Channel A	Channel B
$b_F = 1$ in.	$b_F = 1$ in.
$b_W = 2$ in.	$b_W = 2$ in.
$t_F = 0.10$ in.	$t_F = 0.20$ in.
$t_W = 0.10$ in.	$t_W = 0.10$ in.

Solution for Channel A

$$\frac{b_W}{b_F} = \frac{2}{1} = 2$$

$$\frac{t_W}{t_F} = \frac{0.10}{0.10} = 1$$

$k_F = 0.730$ (read from fig. 3)

$E = 10.66 \times 10^6$ lb. per sq. in.

$\mu = 0.3$

From equation (10)

$$\frac{f_{cr}}{\eta} = \frac{0.730 \times \pi^2 \times 10.66 \times 10^6 \times (0.10)^2}{12\ (1 - 0.3^2)\ \ (1)^2} = 70{,}330 \text{ lb. per sq. in.}$$

From the solid curve of figure 5

$f_{cr} = 33{,}700$ lb. per sq. in.

Solution for Channel B

$$\frac{b_F}{b_W} = \frac{1}{2} = 0.5$$

$$\frac{t_W}{t_F} = \frac{0.10}{0.20} = 0.5$$

$k_W = 6.56$ (read from fig. 4)

$E = 10.66 \times 10^6$ lb. per sq. in.

$\mu = 0.3$

From equation (11)

$$\frac{f_{cr}}{\eta} = \frac{6.56 \times \pi^2 \times 10.66 \times 10^6 \times (0.10)^2}{12\ (1 - 0.3^3)\ \ (2)^2} = 158{,}000 \text{ lb. per sq. in.}$$

From the solid curve of figure 5

$f_{cr} = 38{,}600$ lb. per sq. in.

CONCLUSIONS

1. The critical compressive stress at which cross-sectional distortion occurs in a thin-wall column of channel section or Z-section is given by either of the following equations:

$$f_{cr} = \frac{\eta\, k_F\, \pi^2\, E\, t_F^2}{12\,(1 - \mu^2) b_F^2} \qquad f_{cr} = \frac{\eta\, k_W\, \pi^2\, E\, t_W^2}{12\,(1 - \mu^2) b_W^2}$$

where

E and μ are Young's modulus and Poisson's ratio for the material, respectively.

b_F and b_W, the width of the flange and the web, respectively.

t_F and t_W, the thickness of the flange and the web, respectively.

k_F and k_W, nondimensional coefficients read from figures 3 and 4, respectively.

η, a factor taken so that ηE gives the effective modulus of the flange and web at stresses beyond the elastic range.

2. At stresses beyond the elastic range, the value of the effective modulus ηE for local buckling of thin-wall columns of channel section and Z-section will depend upon tests. In the absence of such tests, however, it is reasonable to assume that η is a function of τ, where τE is the effective modulus of an ordinary column at stresses beyond the elastic range. A careful study of the theory and such experimental data as are available indicates that it is conservative to assume

$$\eta = \frac{\tau + 3\sqrt{\tau}}{4}$$

provided that τ is evaluated by use of the accepted column curve for the material.

Langley Memorial Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., July 11, 1939.

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TABLE I

Calculated Minimum Values of k_F

by the Energy Solution

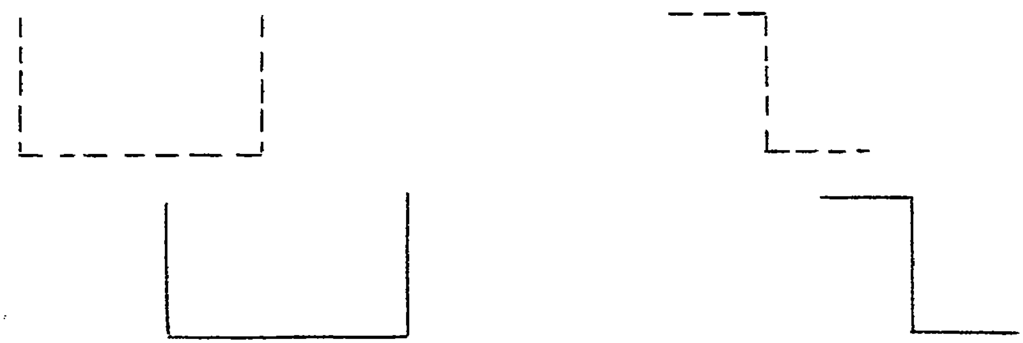
$\frac{b_W}{b_F}$ \ t_W/t_F	k_F 0.5	k_F 1	k_F 2
0	1.288	1.288	1.288
.200	-	1.111	-
.400	.695	-	1.234
.600	-	.962	-
.800	.621	-	1.204
1.000	-	.892	-
1.200	.576	-	1.193
1.400	-	.836	-
1.600	.528	-	1.188
1.750	.506	-	-
1.800	.499	.770	-
1.825	.493	-	-
1.900	.455	-	-
2.000	.410	.730	1.187
2.200	.338	-	-
2.400	.284	.629	1.188
2.800	.208	.521	1.190
3.200	.159	.423	1.192
3.400	-	-	1.178
3.600	.125	.345	1.103
3.800	-	-	1.021
4.000	.101	.284	.940
4.400	.083	.236	.799
4.800	-	.199	.681
5.200	.059	.170	.587
5.600	-	.146	.508
6.000	.044	.127	.444

TABLE II

Calculated Minimum Values of k_W

by the Energy Solution

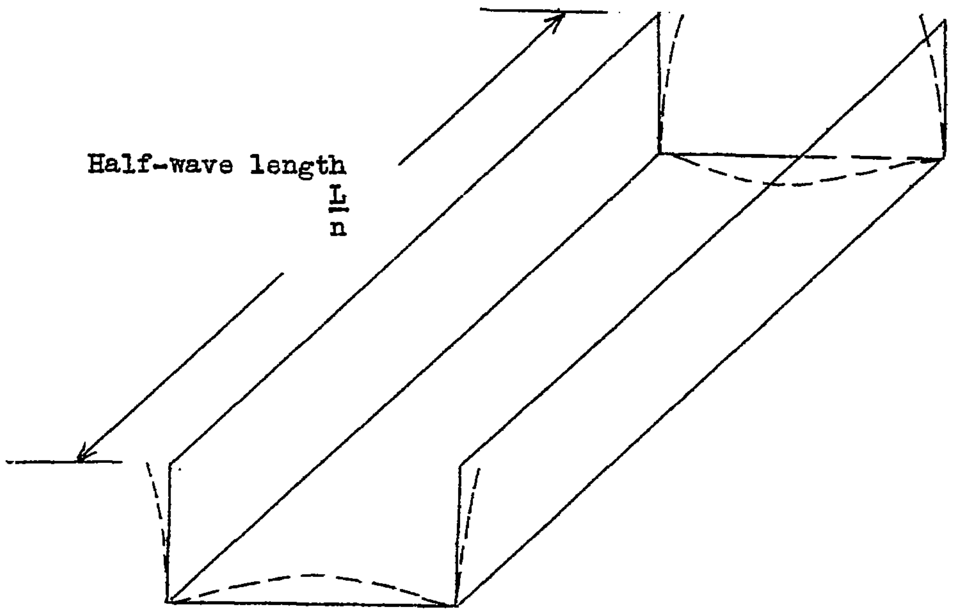
$\frac{b_F}{b_W}$ \ t_W/t_F	k_W: 0.5	k_W: 1	k_W: 2
0	4.000	4.000	4.000
.050	5.457	4.258	4.031
.100	6.020	4.452	4.044
.130	6.188	-	-
.167	6.306	4.585	3.998
.179	-	4.591	3.983
.192	6.381	4.595	3.968
.208	-	4.591	3.922
.227	6.431	4.575	3.865
.250	6.462	4.539	3.762
.263	-	-	3.685
.278	6.493	4.467	3.573
.294	-	-	3.405
.313	6.512	4.333	3.052
.357	6.532	4.081	2.332
.417	6.539	3.625	1.711
.455	6.552	-	-
.500	6.563	2.921	1.187
.526	6.564	-	-
.548	6.567	-	-
.556	6.467	2.496	-
.571	6.204	-	-
.625	5.409	-	.761
.714	-	1.638	-
.833	3.316	-	.429
1.000	-	.892	-



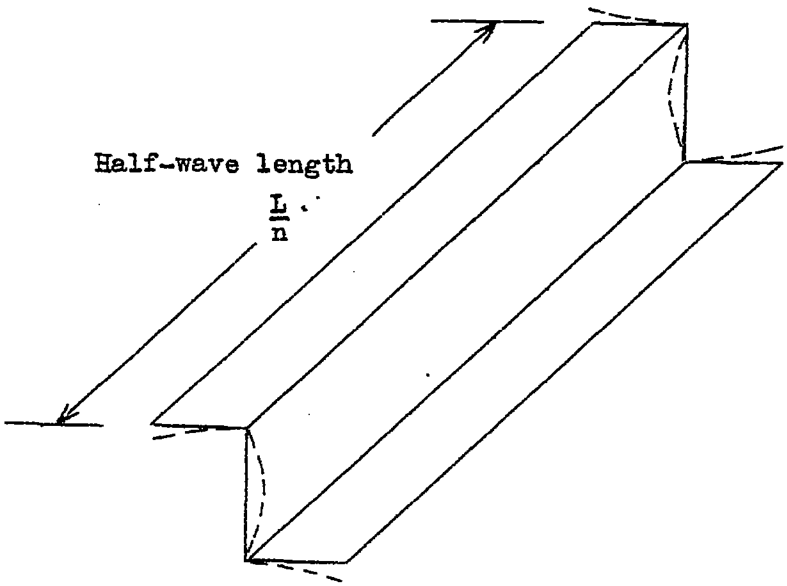
(a) Translated

(b) Translated and rotated

Figure 1.- Displacements of the cross section in primary, or general, failure of a column.



(a) Channel section



(b) Z - section

Figure 2.- Displacements of the cross section in secondary, or local, failure of a column.

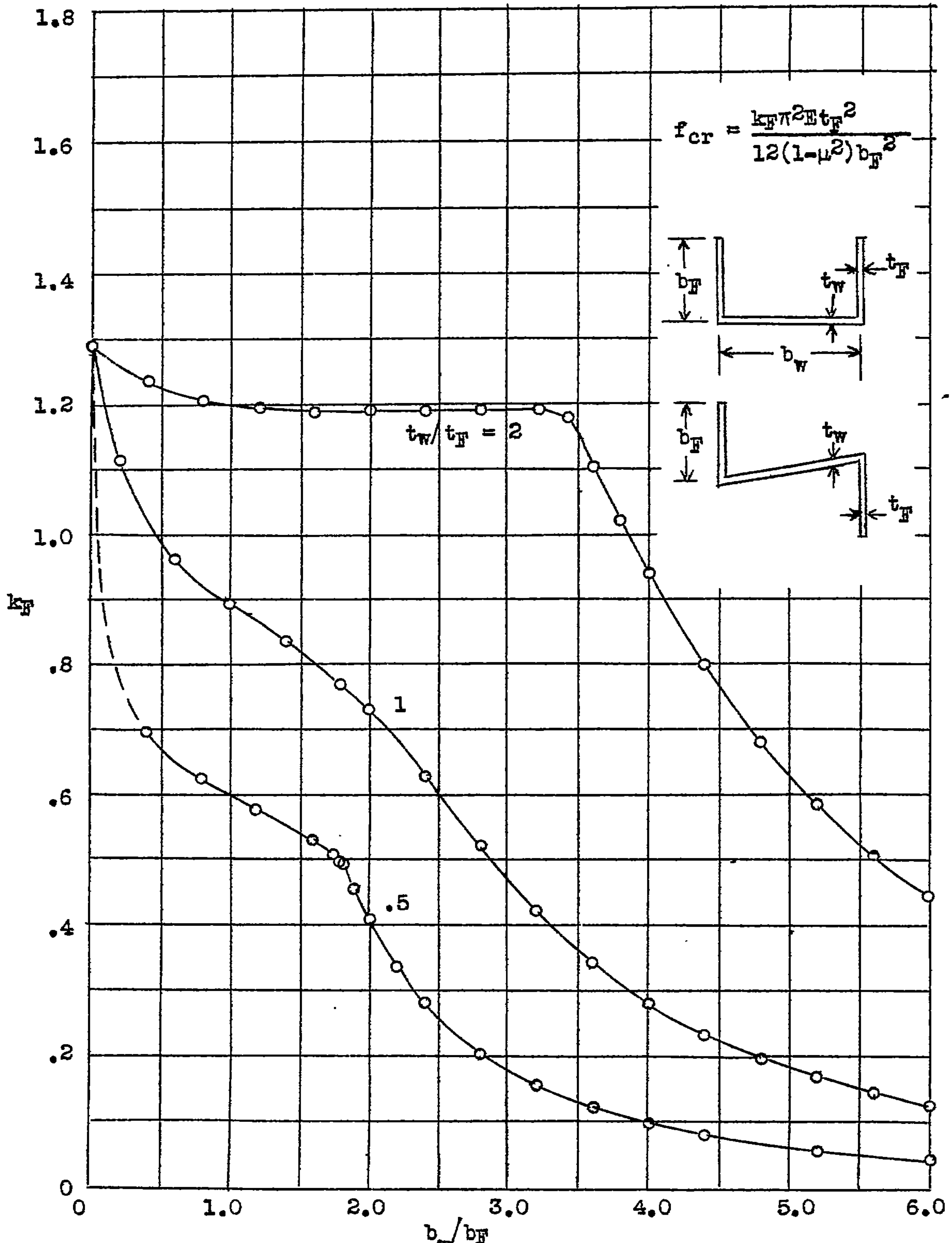


Figure 3.- Minimum values of k_F for centrally loaded columns of channel section and Z - section (μ = 0.3).

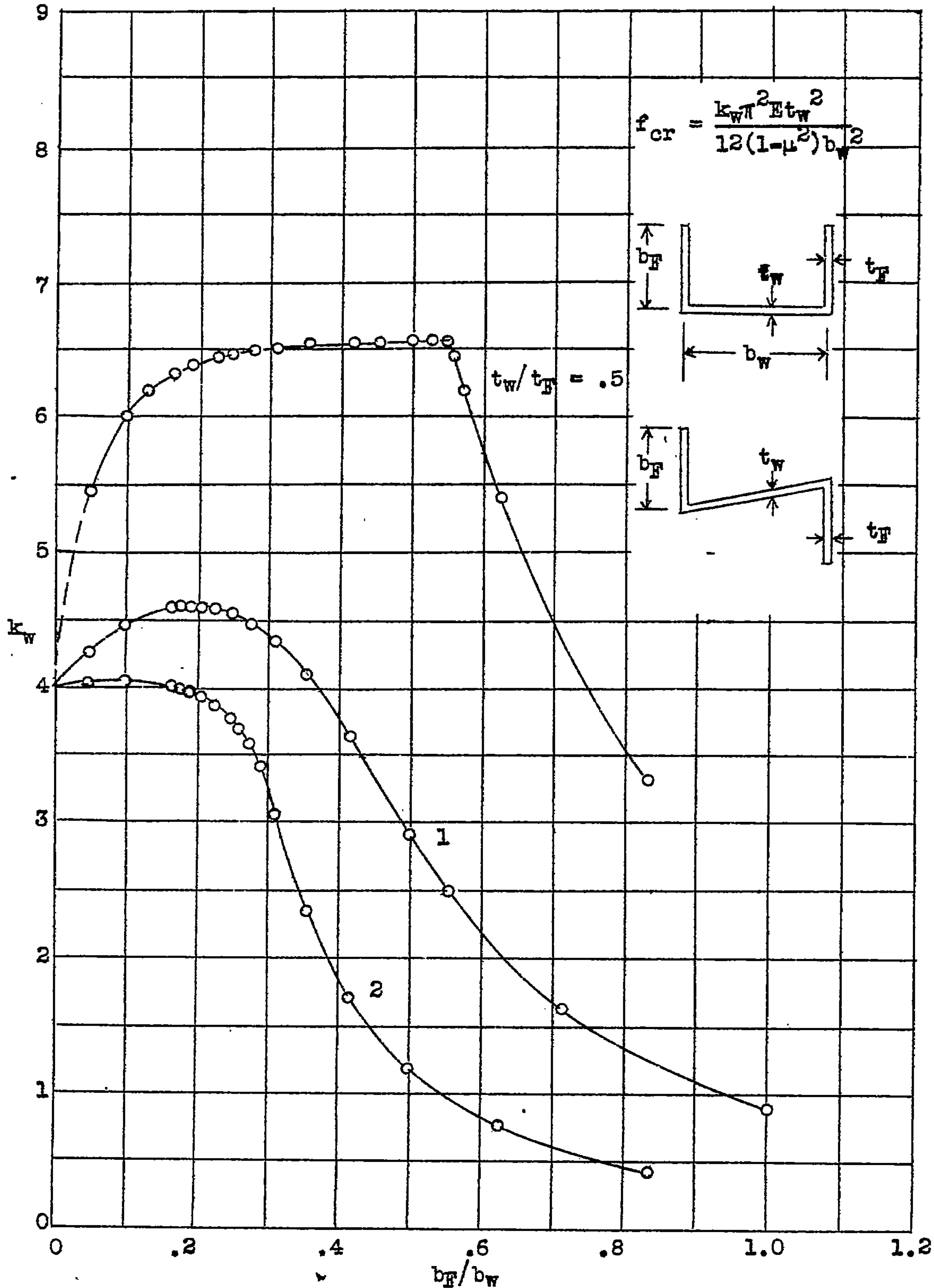


Figure 4.- Minimum values of k_w for centrally loaded columns of channel section and Z- section (μ = 0.3).

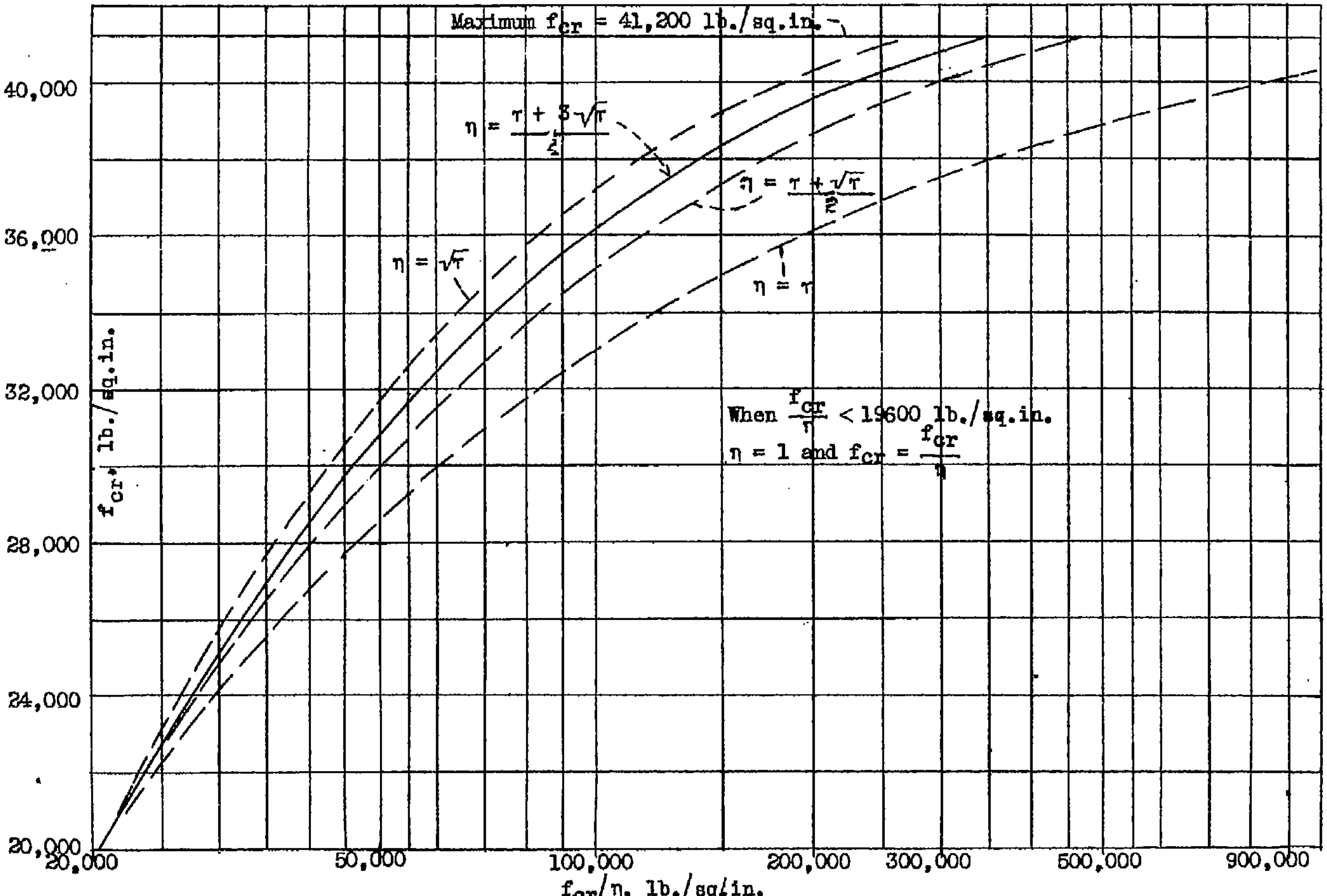


Figure 5.- Variation of f_{cr} with f_{cr}/η for 24ST aluminum alloy.